Exam

Autumn 2019

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Short questions.

   (a) "You have taken a game theory class and have learned that the Nash Equilibrium of the Beauty Contest Game is to say 0. Now you have the chance to play the game in Berlingske with the Danish population." Should you say "0" to win the game? Explain in 2-3 sentences.

   Solution: The Nash equilibrium is 0 under the assumption that players are fully rational. In class we have called this "k-level reasoning". Level zero would mean that they just pick a random number between 0 and 100. At level 1 they take level 0 behavior into account and so forth. In the real world most people reason on level 1 or 2. So you should take that into account and rather choose whatever number corresponds to level 1 or 2 to win.

   (b) Two identical firms compete in quantity in a market. Explain the consequences of changing from the static game (Cournot) to the dynamic game (Stackelberg): who wins, who loses and why.

   Solution: The leader gains and the follower loses in Stackelberg with respect to the profits they would obtain in Cournot. The reason is that the reaction functions indicate that, as the rival's quantity increases, one must reduces its own. The leader moves first and can increase its production knowing that the rival will reduce his. Consumers are better off in the Stackelberg competition.

   (c) In a finitely repeated game one can always find a SPNE in which the payoffs are Pareto optimal. Is that statement true or false? Why?

   Solution: False. If there is only one NE in the static game, the repetition of the game will only have the repetition of the NE in every subgame as the SPNE. The NE does not need to be Pareto optimal as we saw, for instance, in the prisoners' dilemma game.

   (d) Show how the phenomena of overfishing can be represented as a Prisoners' Dilemma. (hint: set up the game with two players, each of which can undertake low or high fishing activity). Explain your set-up and the Nash equilibrium.

   Solution:

		P2: High fishing	P2: Low fishing
P1	High Fishing	$1, 1$	$3, 0$
P1	Low Fishing	$0, 3$	$2, 2$

The sustainable fishing catch is higher when both nations undertake low fishing activity. However, there is then an incentive for both to increase fishing. In fact, high fishing is a dominant strategy for both players. We therefore, end up with the worst outcome.

(e) Distinguish simultaneous-move games and dynamic games in terms of the types of information we have discussed in class. Explain why in dynamic games Nash equilibria may not be subgame perfect. Using examples, show how non-credible threats are ruled out using backward induction.

Solution: In dynamic games information will be revealed by one player making a move before another player. In simulteneous games, information is imperfect. Nash equilibria are not always subgame perfect because the concept on Nash equilibrium does not take into account the temporal dimension of the decision process in a dynamic game. So, menaces that at the beginning of the game might be used to threaten the opponent, would never be used when the time comes. Subgame perfection can therefore be considered a refinement applied to the Nash equilibrium concept that selects equilibrium strategies that are credible in every subgame of the whole game. The market entry game is a good example to show the way in which backward induction works.

2. Find all Nash equilibria (pure and mixed) in the following game:

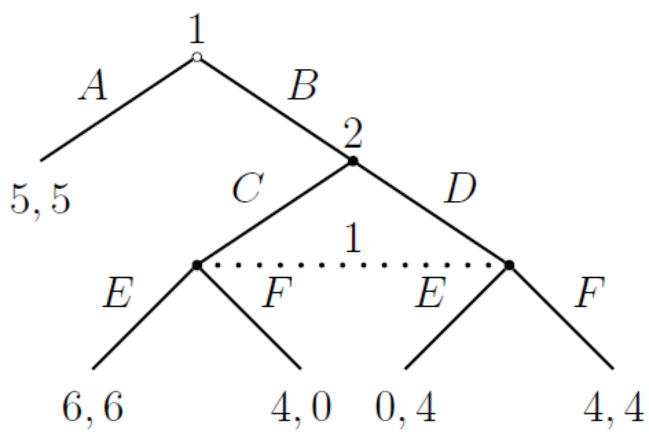
		P2 X	P2 Y	P2 Z
P1	A	$2, -1$	$4, 2$	$2, 0$
P1	B	$3, 3$	$0, 0$	$1, 1$
P1	C	$1, 2$	$2, 8$	$5, 1$

Solution: There are three equilibria. (B,X) and (A, Y) are PSNE and

$$(\frac{1}{2}, \frac{1}{2}, 0), (\frac{4}{5}, \frac{1}{5}, 0) \tag{0.1}$$

is a mixed-strategy Nash equilibrium.

(a) Have a look at the following game:



(b) Find all the pure-strategy Nash equililibira of this game.

> **Solution:** (BE,C), (AE,D), and (AF,D)

(c) Which of these are sub-game perfect?

> **Solution:** (BE,C) and (AF,D)

(d) Look at the SPNE and try to come up with an argument for why one of them should be eliminated. Remember, players believe that other players are rational. Past actions need to be rational and future actions need to be rational. Briefly explain your argument.

> **Solution:** For player 1, note that BF is dominated by AF (or AE). Thus, if player 2 believes that 1 is rational, she would never expect 1 to play F in the subgame, if it is reached. So when player 2 is at her decision node, she ought to believe that 1 chose B and will go on to choose E. In that case, 2 would choose C. Therefore, the (AF,D) equilibrium should be eliminated.

3. A first edition book by Kirkegaard is being auctioned off. The auction is held as a first-price sealed bid auction. Jens values the book at v1 and Mikkel at v2. Their values are distributed independently and identically distributed" with uniform distribution over [0, 1] $v_i, i = 1, 2$

(a) Assume that both bidders use linear strategies, i.e. $b_i(v_i) = a_i, v_i, i = 1, 2$, where a_i is a positive constant. Find the equilibrium bidding strategies (i.e. the values of $a_1; a_2$).

> **Solution:** Jens, observing value v_1 and submitting bid b_1 will have a pro
> fit of $v_1 - b_1$ provided he wins the auction (it is easy to see that ties happen with probability 0 under independent continuous valuations and linear strategies). Expected payoff is thus:
>
> $$E(\Pi_1 \mid v_1, b_1) = (v_1 - b_1)P(win \mid b_1) = (v_1 - b_1)P(b_1 > b_2) = (v_1 - b_1)P(v_2 < \frac{b_1}{a_2})$$
>
> Clearly, there is no reason for Jens to ever bid higher than the highest possible bid of Mikkel, which is a_2. Then, given the distribution of v_2 we have:
>
> $$E(\Pi_1 \mid v_1, b_1) = (v_1 - b_1)\frac{b_1}{a_2}$$
>
> Which for any v_1 is maximized at $b_1 = v_1/2$. Therefore the optimal strategy of Jens is $b_1 = v_1/2$ so $a_1 = 1/2$ and, by symmetry, $a_2 = 1/2$.

(b) Now assume that the seller uses a descending auction format: the price goes down from one and the first buyer to stop it buys the object at this price. Find the equilibrium bidding strategies.

> **Solution:** Descending auctions are strategically equivalent to the first-price-auction, which means there is a one-to-one mapping between the strategy sets and the equi-

libria of the two games. In descending auctions you do not get any useful info (until it is too late) and you pay the price you name so it is exactly like first-price sealed-bid.

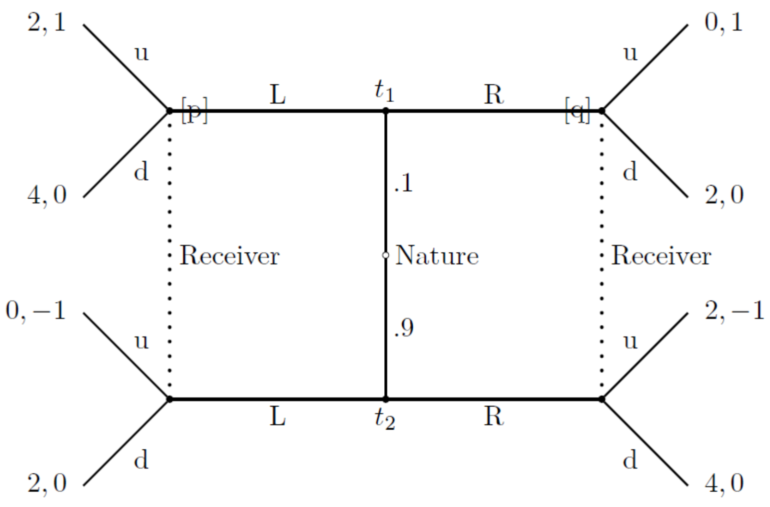
(c) Now assume that Mikkel is entitled to buy at the price offered by Jens, i.e., Jens submits his bid of choice, say: b, and Mikkel either buys at this price or steps out (in which case Jens purchases the object at price b). Find optimal strategies for both players.

Solution: It is a dynamic game. For Mikkel it is easy he should buy if his value is higher than the bid of Jens, $v2 > b$, he is indifferent in the improbable case of $v_2 = b$ and he should not buy if $v_2 < b$. So Jens submitting a bid $0 < b < 1$ knows he will get the object with probability b. In order to maximize the expected profit $(v_1 - b)b$ he must choose $b = v_1/2$, as in part a).

(d) Compare the expected revenues obtained by the seller under point a) and c) Does the Revenue Equivalence Theorem result hold? If not, why not?

Solution: In a) the seller in equilibrium obtains 1/2 of the higher value, which is 1/3 in expectation. In c), the seller obtains 1/2 of the value of Buyer 1, which is 1/4 in expectation. The revenue equivalence theorem does not hold because the efficiency assumption is not satisfied under the auction format in c). The object could go to buyer 2 although he has a lower valuation for example when $v_1 = 0.6$, $v_2 = 0.4$. Buyer 1 will submit b=0.3 and will not get the object despite having higher value.

4. Consider the following game G':



(a) Is G' a dynamic or a repeated game?

Solution: By definition, G is a dynamic game, where the Sender chooses a message and the Receiver then responds with an action.

(b) Find a separating equilibrium in G' , and find a pooling equilibrium where both sender types play L. Show the steps of how to get to the solution. Explain your process.

Solution: Separating equilibrium: (LR, ud, p = 1, q = 0). Pooling equilibria: (LL, du, p = 0.1, $q \geq 1/2$).

(c) Describe a hypothetical real-world strategic situation that could correspond to G', and explain why this is the case (3-4 sentences).

Solution: Possible solution: The Sender could be a start up with two possible levels of skill in dealing with competition (t1 is low-skill, t2 is high-skill). Each message could be a possible successful financing round for the start up, where the low-skill (high-skill) start up finds attracting financing R more costly. The Receiver could be a new start up deciding whether to compete (u) or not (d) with the start up in the product market. The analysis above might suggest that the start up attracts venture capital funding that reveals his skill, and the other firm decides to compete if and only if this funding is revealed to be low (separating equilibrium). In the pooling case both types do not attract any venture capital funding and thus, the receiver doesn't learn about the type before deciding to compete.

(d) Imagine now messages have no direct effect on payoff. Only sender type and action of the receiver matter. What do we call those types of games? What would need to change in the description of the game? You can draw a new game as an example.

Solution: Cheap Talk games. The messages should no longer matter. Only types and actions of the receiver matter for payoff. For example, for Type t1 and action u, the payoff should be 2,1 in both cases, regardless of message. For t1 and action d, it could be 4,0 regardless of message. Same reasoning for type 2.